Re-Optimizing Food Systems
A Dynamic Analysis of Production, Sustainability, and Equity

by

Anthony Artino and Sarah Decipeda

A Research Project

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Under the supervision of
Dr. Morley Davidson

Department of Mathematical Sciences
Kent State University

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Abstract

Our food system prioritizes production efficiency and profit above the health of the environment and equitable distribution. Because of this, the planet decays and people needlessly starve. Our goal is to improve the food system by modeling it in Python using four differential equations: food production (F), natural resource health (N), economic capital (C), and equity (Eq). We apply the system of equations at the state level by comparing Louisiana, Connecticut, and New Hampshire, which have high, average, and low rates of food insecurity, respectively. Using dynamic feedback loops, we impose appropriate policies to adjust and improve the states' systems. While the model is useful for observing and predicting the system dynamics, it cannot perfectly account for the delays that occur when enacting policies due to real-world behavior. We must consider a realistic timeframe for political action and implement the delay ourselves. A program can immediately correct a problem, but a politician cannot.

Contents

Introduction

The instability of our food system on a global level damages both environmental health and the well-being of people all around the world. Food production causes 26% of all greenhouse gas emissions, uses 70% of freshwater, and drives deforestation and loss of biodiversity [15]. On top of environmental destruction, an estimated 673 million people faced hunger, and 2.3 billion people were food insecure in 2024. Food insecurity occurs when a person does not have access to the necessary amount of nutritious food for growth, development, and leading a healthy life [14].

Even a nation as wealthy and developed as the United States has prevalent food insecurity. In 2024, 18.3 million households experienced some level of food insecurity. Most of these households had low security, meaning the household members received enough food via federal or community programs, but meals were less varied and nutritious. However, 7.2 million of these households had very low security. One or more members had disrupted eating patterns or skipped meals altogether due to a lack of resources. This includes 751,000 children who experienced severe food insecurity [17].

Despite producing more than enough food to feed its population, over 33% of the food from the U.S. food system is wasted. This waste acts as a double-edged sword for ecological damage. The environmental costs and resources used to produce, transport, and store the food have already been expended, and sending the wasted food to be landfilled or incinerated generates more carbon dioxide emissions and methane that harm the atmosphere [1]. The U.S. Environmental Protection Agency estimates that, in addition to the landfill emissions, food waste creates an amount of greenhouse gas emissions equivalent to 42 coal-fired power plants [5].

Our project implements hypothetical government policies that place more importance on the protection and regeneration of natural resources as well as the equitable distribution of food. We apply these policies to states with low, average, and moderate food insecurity, compared to the national average of 13.7% [17]. First, we observe the states' individual food systems without government intervention. Then we examine the effects of immediate intervention when a critical value in food production, natural resource availability, or capital efficiency is reached. Finally, we use the results from the previous model to set a realistic timeline for political action. Under the legislation, more people will have access to food, and the environment will be healthier.

Chapter 1

Mathematical Foundations of the Food System

In this chapter, we find the foundational ordinary differential equations (ODEs) working with the food system. This model treats agriculture as a highly dynamic system rather than a purely economic venture. We define four linked state variables: Food Production Dynamics (F), Natural Resource Health (N), Economic Capital (C), and an Equity Index (Eq). This system relies on the dynamic policy parameters that are controlled by government intervention. These specific parameters are (u_p), which represents the effort dedicated to aggressive food production; (u_s), which defines the total percentage of the capital allocated to sustainability; and finally, (u_e), which dictates the capital allocated to equity and food distribution. Thus, balancing these equations allows us to simulate the long-term viability of the food architecture.

1.1 Food Production Dynamics

First off, the main objective of the food subsystem is to generate agricultural biomass to meet the caloric and nutritional consumption demands of the population. Food production requires both natural resources as well as financial backing, as it continues to battle any sort of systemic waste and human consumption.

$$\frac{dF}{dt} = \alpha \cdot C \cdot N \cdot u_p - \min(\beta \cdot P, F) - \gamma \cdot F \tag{1}$$

1.1.1 Mathematical Formulation

In this case, the parameter α represents the technological/infrastructural efficiency of converting the natural resources as well as the economic capital into a consumable food supply. We represent the available wealth utilized to purchase equipment and labor as economic capital (C). The Natural Resource Health (N) dictates the current state of the environment, including soil fertility and water availability. The Production Policy Lever (u_p) signifies the total effort of the system that is then dedicated to aggressively producing food, where it ranges from 0 to 1.0. The parameter β defines the per capita consumption rate of the population, which we set to 0.05. We define the number of each individual to heavily rely on the food system as the total population (P). Finally, the decay rate (γ) accounts for the natural spoilage of food as well as the systemic waste within the overall supply chain.

This equation is used to model food as a transient resource. The positive growth term mathematically will link the economy to the environment, which shows that food production will end up failing without both money and healthy land. Now, if the instantaneous production rate fails to outpace consumption and decay, then the total food biomass will drop below the critical demand threshold, which will trigger a systemic crisis.

1.1.2 Computational Results and Discussion

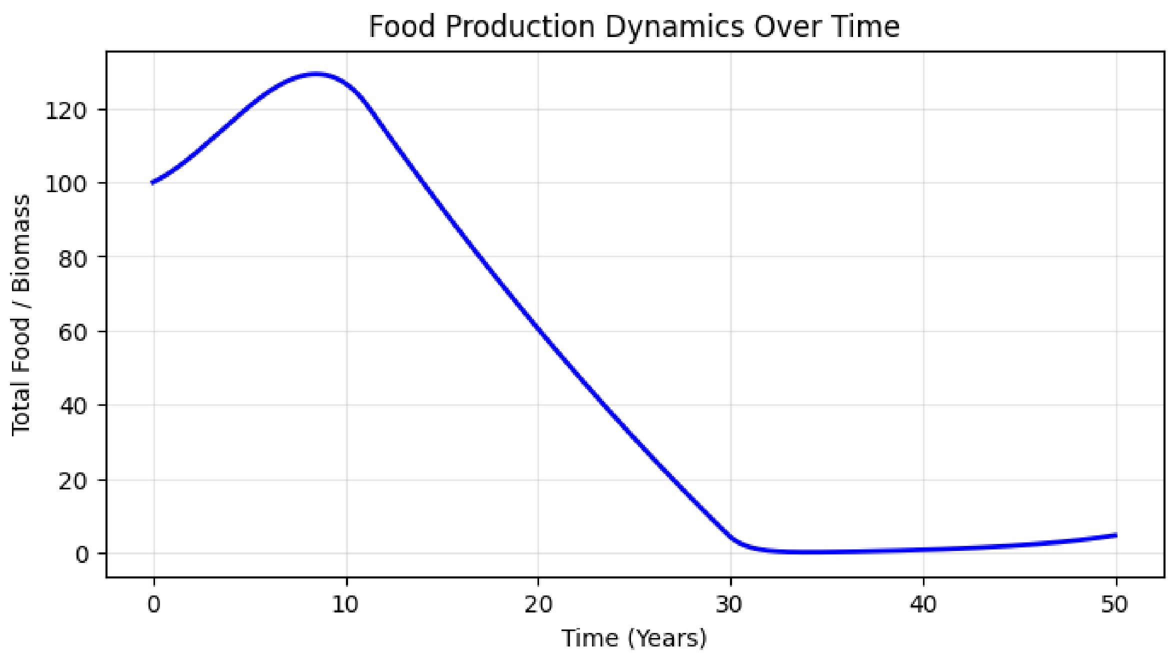

Figure 1.1

Food production rises and comfortably feeds a population of 100, but shows a linear decline after about 10 years. Production drops to zero and stays there until Year 40, when it slowly begins to rise.

1.2 Natural Resource Health

The health of the environment acts as the limiting biological factor in this bioeconomic system. The model accounts for the natural regeneration of the earth, the degradation caused by aggressive industrial farming, and the active repair funded by government sustainability initiatives.

$$\frac{dN}{dt} = r \cdot N \left(1 - \frac{N}{K}\right) - \delta \cdot F \cdot u_p + \sigma \cdot u_s \tag{2}$$

1.2.1 Mathematical Formulation

The natural regeneration rate (r) indicates the overall speed at which the environment tends to heal itself. The carrying capacity (K) establishes the maximum potential health index of the natural environment and stops at 1.0. The Degradation Rate (δ) shows the specific damage aggressive agricultural extraction inflicts upon the soil and water. The Harvesting Intensity term ($F \cdot u_p$) captures the volume of food that is farmed and extracted from the earth. The Sustainability Efficiency (σ) evaluates the total effectiveness of environmental investments that restore natural health. Finally, the Sustainability Policy parameter (u_s) defines the percentage of the system's economic capital actively redirected toward environmental preservation.

We have a modified logistic growth function to model the environment since nature reaches a state of equilibrium when left undisturbed. However, modern agriculture prevents this equilibrium. Furthermore, the extraction penalty tends to capture the ecological footprint of farming. As the system produces more food, the environment degrades a lot faster. Hence, the sustainability addition allows the model to test whether the financial investments in green technology successfully offset the damage caused by industrial harvesting.

1.2.2 Computational Results and Discussion

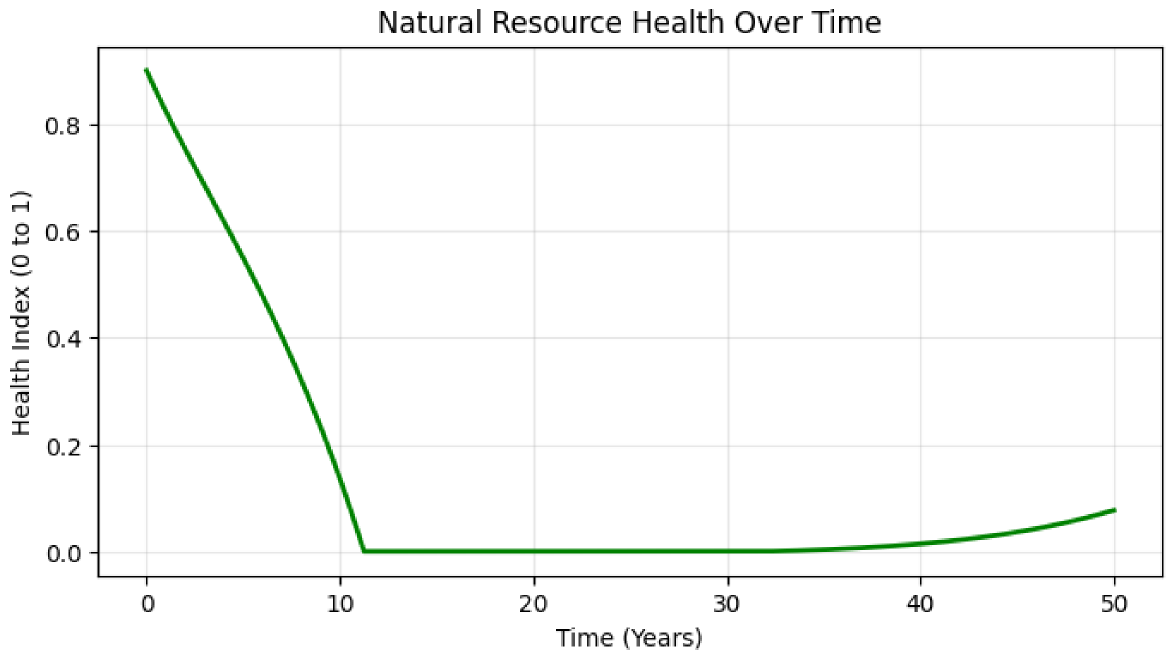

Figure 1.2

We initially see a high index, indicating a thriving environment. However, food production exhausts natural resources in 10 years, leaving the health index at 0.0 until about Year 35. With no food production, the environment slowly regenerates.

1.3 Economic Capital

Economic Capital drives the entire system. In this case, it funds agricultural operations, supports the environmental recovery, and also pays for equity distribution. The capital variable operates as the GDP or total available wealth of the modeled state.

$$\frac{dC}{dt} = p_{margin} \cdot \min(\beta \cdot P, F) - c \cdot F - (u_s + u_e) \cdot C \tag{3}$$

1.3.1 Mathematical Formulation

The profit margin (p_{margin}) works as a multiplier that represents the revenue generated per unit of food successfully sold and consumed by the population. The Total Consumption term ($\beta \cdot P$) determines the precise amount of food consumed by the total population, representing the market demand that drives the revenue. The base production cost (c) establishes the financial cost required to grow and process a single unit of food. The Total Food Produced (F) sums up the aggregate volume of agricultural output, and finally, the intervention costs ($u_s + u_e$) make up the combined percentage of capital which the government redirects from pure economic growth to fund sustainability and equity programs.

This equation shows the main reality of financial boundaries. For instance, a system isn't infinitely funding social and environmental programs without a certain consequence taking place. The economic capital also grows strictly through the sale of consumed food, but will continue to drain due to the absolute cost of mass production. Allocating resources to sustainability or equity directly drains capital. This creates a mathematical tension where maximizing profit will directly conflict with maximizing environmental health as well as social fairness.

1.3.2 Computational Results and Discussion

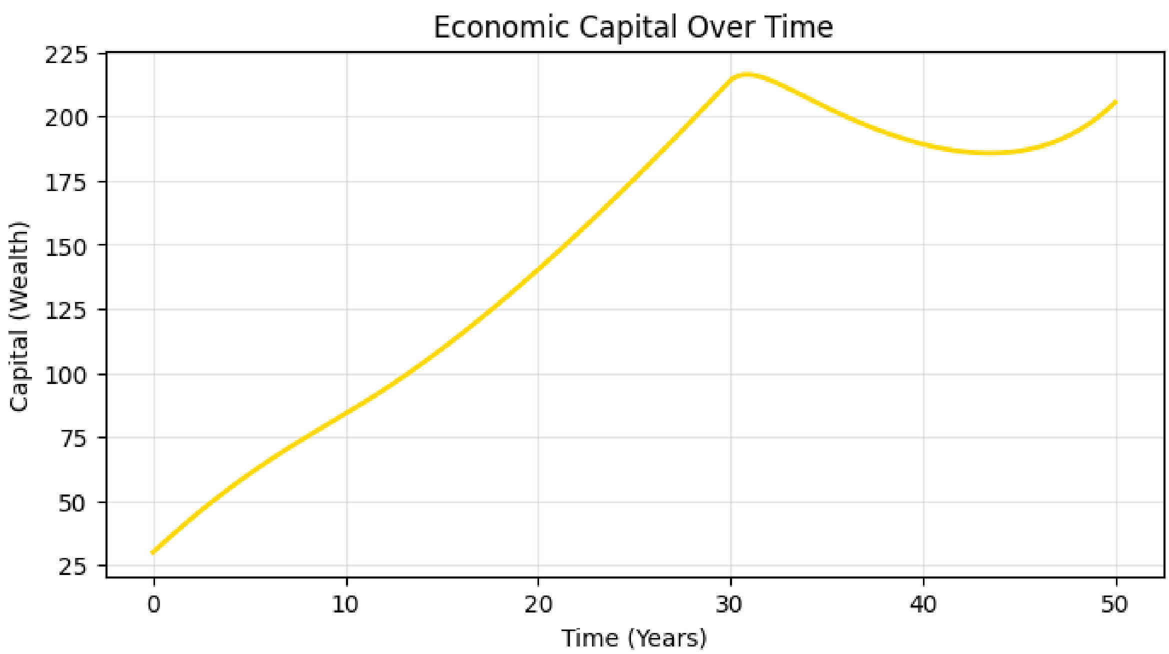

Figure 1.3

Capital rises as long as there is food being produced and sold. As food production declines, the total cost of production lowers as well, and available food is still sold. At Year 30, food production reaches zero. There is no more food to sell, so capital declines. When natural resources regenerate, and food production picks up again in Years 40-50, capital also rises.

1.4 Equity Index

Finally, our last variable measures the fairness of food distribution and general accessibility. For instance, high food production doesn't inherently mean the population eats if the distribution ends up remaining highly unequal. The Equity Index tracks the societal well-being of the system.

$$\frac{dEq}{dt} = k_1 \cdot u_e + k_2 \cdot \max(0, F - D) - \mu \cdot Eq \tag{4}$$

1.4.1 Mathematical Formulation

The Equity Investment Impact (k_1) serves as an increasing coefficient that determines how effectively the overall government funding improves social fairness. The Equity Policy parameter (u_e) initiates the percentage of capital actively spent on distribution infrastructure, such as food banks or subsidies. The Surplus Impact (k_2) represents how an abundance of food naturally decreases the prices and improves the general accessibility. The Food Surplus term ($\max(0, F - D)$) calculates the total amount of food that is being produced, which strictly exceeds the baseline population demand (D). If the demand goes unmet, this term safely zeroes out to prevent any sort of equity growth. Finally, the Natural Equity Decay (μ) represents the rate at which the societal inequality will naturally widen as long as it's left completely unaddressed by policy or surplus.

We use this equation to address the human element of food insecurity, which often stems from a poor distribution rather than a pure scarcity. This model states that the equity will improve by active financial intervention or a massive biological surplus that makes food cheap and abundant. Without constant effort or surplus, the decay term will guarantee that the system regresses towards inequality, which forces governments to proactively manage distribution.

1.4.2 Computational Results and Discussion

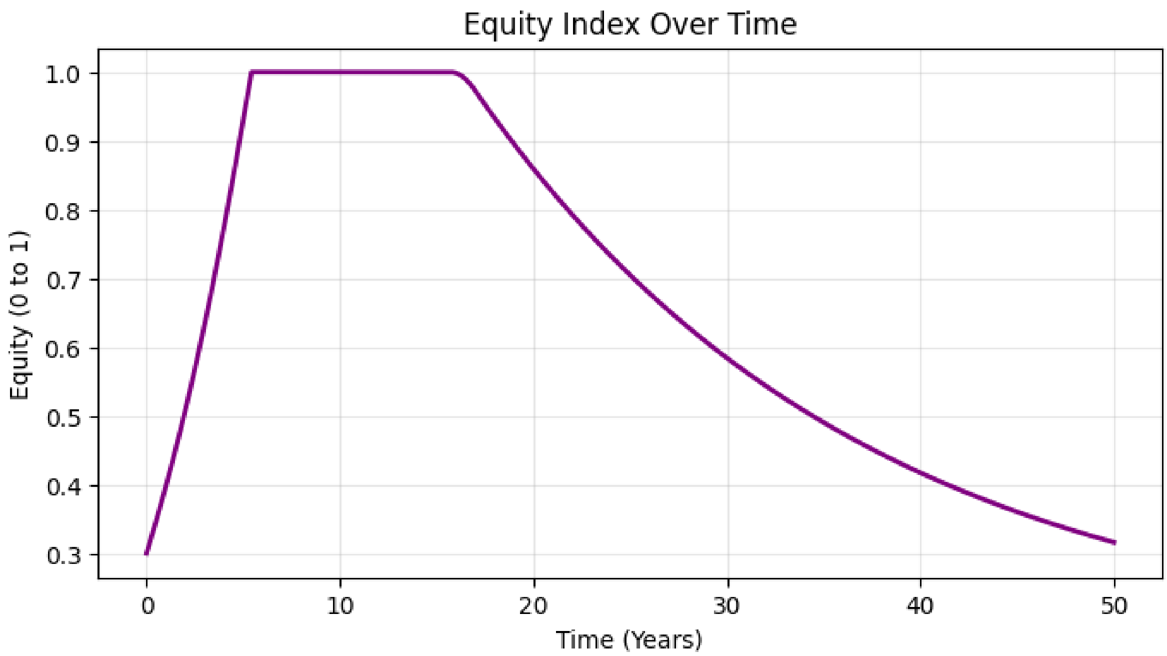

Figure 1.4

With ample food production, the system feeds everyone, causing equity to quickly rise and stay at 1.0 until about Year 16. When production declines, fewer people have access to food. Thus, the curve declines for the rest of the time period. The slight increase in food production in the later years is not enough to raise equity.

Chapter 2

Dynamic Feedback: Immediate Intervention and Political Delay

As we continue on, our baseline models established in the previous chapter reveal a critical flaw in static governance. This flaw consists of systems that are optimized purely for profit, inevitably pushing the environment toward ecological collapse as well as the population toward severe inequality. To prevent these outcomes, the mathematical architecture must incorporate some dynamic feedback. We define a crisis-response framework where the system constantly monitors two critical thresholds. If the available food drops below 110% of the basic demand threshold ($D \cdot 1.1$), if the natural health index falls below a critical ecological marker ($N < 0.4$), or if economic capital drops below a critical minimum ($C < 20.0$), the system identifies a systemic crisis. Upon detecting this crisis, the state shifts its policy instruments to prioritize recovery and distribution over raw economic growth.

2.1 Immediate Intervention

With immediate intervention, this scenario assumes a perfectly efficient government that implements policy shifts the precise moment a crisis occurs. This theoretical model serves as a mathematical benchmark to help measure the maximum possible effectiveness of the control variables. For instance, when the system detects a shortage or a possible severe environmental degradation, it can instantly reallocate the state capital. Furthermore, rather than applying a blanket policy, the government implements state-specific interventions tailored to the exact nature of the crisis, whether that requires prioritizing environmental bailouts, emergency food subsidies, or preventative maintenance.

Regional Application and Numerical Analysis

Now, we can apply this immediate intervention model across three distinctive regional profiles to demonstrate how different baseline conditions require the timing and mathematical impact of the policy shift.

Louisiana (The Immediate Social Crisis)

Louisiana serves as the high-insecurity profile. It first begins with the critical deficits in food ($F_0 = 40$) and capital ($C_0 = 10$). Due to the initial food supply sitting far below the demand threshold (80), the system triggers an immediate intervention policy right at the moment where the simulation starts. We calculate the new trajectory for the Equity Index by using the crisis policy parameters ($u_p = 0.8$, $u_s = 0.05$, $u_e = 0.05$):

$$\begin{aligned}\frac{dEq}{dt} &= k_1 \cdot u_e + k_2 \cdot \max(0, F - D) - \mu \cdot Eq \\ &= 0.7(0.05) + 0.005(0) - 0.05(0.1) \\ &= 0.035 - 0.005 \\ &= 0.03 \text{ index units per year}\end{aligned}$$

By redirecting the 5% of the capital directly into the food distribution programs, the policy forces the equity index to grow rapidly by instantly mitigating the social crisis. However, this intervention does limit the economic capital. The intervention slashes the capital growth rate dropping from 10.1 to 9.5 units per year, mathematically proving the inherent cost of recovery. Thus, stabilizing the population directly drains the state's wealth.

Connecticut (The Delayed Capital Drain)

Connecticut represents an average insecurity profile. It possesses high initial capital ($C_0 = 85$) and then equates to the initial food ($F_0 = 110$). However, it does lack the overall agricultural efficiency ($\alpha = 0.25$). Since its initial food sits above the threshold, the state ends up avoiding any sort of immediate intervention. In this case, we calculate its baseline food production trajectory:

$$\begin{aligned}\frac{dF}{dt} &= \alpha \cdot C \cdot N \cdot u_p - \min(\beta \cdot P, F) - \gamma \cdot F \\ &= 0.25(85)(0.75)(0.7) - 0.05(100) - 0.04(110) \\ &= 11.15 - 5.0 - 4.4 \\ &= 1.75 \text{ food units per year}\end{aligned}$$

Despite its wealth, Connecticut gains 1.75 units of food per year. However, the aggressive farming decays the environment, which initiates the total intervention at Year 22. Once this scenario initiates, the heavy 60% intervention tax ($u_s = 0.50, u_e = 0.10$) completely overwhelms the state's moderate capital generation, forcing Connecticut into a much more deliberate economic recession to guarantee its population remains fed.

New Hampshire (The Ecological Mask)

New Hampshire represents the low-insecurity profile characterized by highly efficient farming ($\alpha = 0.28$) as well as a much massive initial food surplus ($F_0 = 130$). For instance, New Hampshire tends to remain in a profit-driven state for the longest duration, mainly due to its surplus hiding the underlying ecological damage. We calculate the natural degradation under its baseline policy:

$$\begin{aligned}\frac{dN}{dt} &= r \cdot N\left(1 - \frac{N}{K}\right) - \delta \cdot F \cdot u_p + \sigma \cdot u_s \\ &= 0.25(0.9)(0.1) - 0.0002(130)(0.7) + 0.3(0.02) \\ &= 0.0225 - 0.0182 + 0.006 \\ &= 0.0103 \text{ index units per year}\end{aligned}$$

In this case, we see that the natural regeneration slightly outpaces the aggressive harvesting, as this pushes the health index up by 0.0103 units annually. Over decades, if the environment collapses below the 0.4 threshold, as this initiates the sustainability intervention. By dropping the production effort to $u_p = 0.5$ and then raising the sustainability funding to $u_s = 0.05$, the extraction penalty ends up shrinking while the regeneration term increases as this successfully halts the ecological free-fall before the soil ends up becoming permanently barren.

2.1.1 Computational Results and Discussion

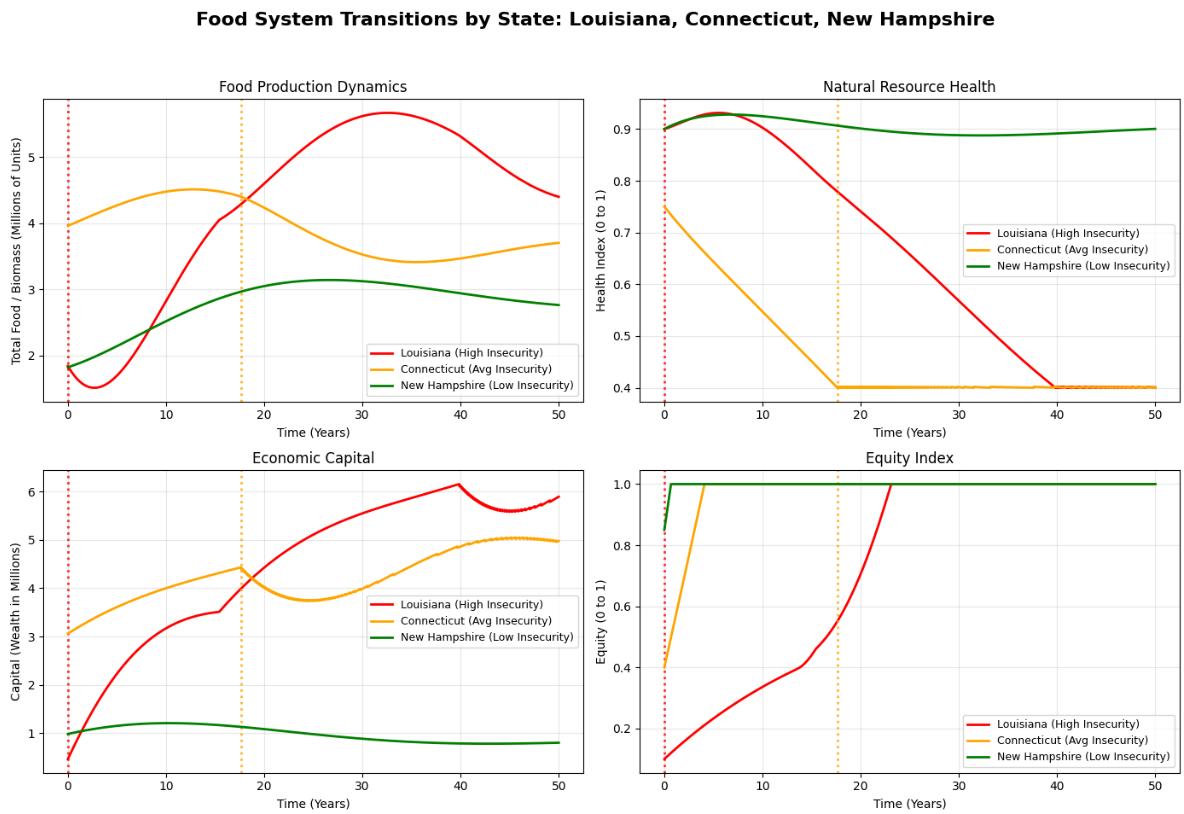

Figure 2.1: Simulation of the four state variables under immediate intervention across Louisiana, Connecticut, and New Hampshire. The system stabilizes quickly, preventing irreversible ecological or social collapse.

High Insecurity: Because initial food levels are already below the threshold, we see the emergency food crisis policy implemented at Year 1. Food production increases drastically, peaking at about Year 33. High production depletes natural resources until the health index reaches its threshold, forcing the environmental policy to begin at Year 40. The policy allows for nature to recover and is then removed. However, the previous "balanced" state pushes the health index back down, initiating the policy again. This feedback cycle causes natural resource health to hover around the threshold level of 0.4. Economic capital rises, but has a dip when food production decreases. Less food is sold, but the cost of production also lowers, so we see capital increase again. Equity steadily increases before spiking to total equity.

Average: Food production rises until it depletes the environment. The state triggers the environmental collapse policy around Year 18, and the health index hovers at 0.4 for the same reason as in Louisiana. Food is still able to be produced, so capital increases after the policy causes an initial decline. Adequate production allows equity to be achieved and maintained.

Low: The policy is never triggered. New Hampshire's already high equity index, abundant natural resources, and low population allow the state to feed itself and maintain a healthy environment and economy without intervention.

2.2 Political Delay

The political delay model introduces the realistic human constraints to the mathematical feedback loop. In this scenario, the governments do not respond to crises that occur instantaneously due to the legislative drafting cycles, public skepticism, and even corporate lobbying. We model this friction as a time-lagged response. The system mathematically identifies a crisis but fails to enact the required policy changes for several years. This delay forces the state variables to continue their destructive trajectories into a danger zone, where recovery becomes way more expensive or biologically impossible.

The Mathematical Cost of Delayed Responses

We assign specific delay intervals based on the urgency and visibility of the crisis in each state as this results in some catastrophic overshoot effects.

Louisiana (6-Year Delay)

Louisiana faces a survival crisis, but the administrative hurdles create a 6-year delay. During this window, the unintervened system drains the food supply, and the equity index drops to zero. The unchecked profit margins inflate the economic capital to $C = 65$. When the government finally imposes the 10% policy tax at Year 6, the massive capital pool results in an enormous absolute financial drain. The capital growth drops to 7.5 units per year. Thus, this proves that delaying the intervention to save money in the short term does make the eventual cost of stabilization exponentially higher.

Connecticut (22-Year Delay)

Connecticut experiences a less visually urgent decline as it results in a 22-year delay. Since the state holds great wealth, the general public does not recognize the rising food insecurity until it ends up becoming a much more dominant, unavoidable voting issue. For instance, during these 22 years, the state uses its overall wealth to import and purchase food at exorbitant baseline costs. By the time the government ends up passing an environmental bailout at year 22, imposing a massive 50% sustainability tax ($u_s = 0.50$), the baseline capital has already been drained by the cost of sustaining an inefficient food system, which leaves the state without the funds required to properly execute the recovery without triggering a deliberate economic recession to save the dying soil.

New Hampshire (30-Year Delay)

New Hampshire faces the most deceptive challenge as it results in a massive 30-year delay. In this case, New Hampshire produces a heavy surplus, so because of this, the population remains well-fed. Because the state's natural regeneration successfully balances out the farming degradation, the environment never collapses. The 30-year delay in this scenario does not result in an ecological free-fall; instead, it represents the time it takes for the government to shift from a profit-driven baseline to a preventative maintenance policy. At year 30, the government proactively adjusts the levers ($u_p = 0.5, u_s = 0.05$) to ensure the health index (N) reaches its maximum carrying capacity before any long-term population growth can destabilize the system.

2.2.1 Computational Results and Discussion

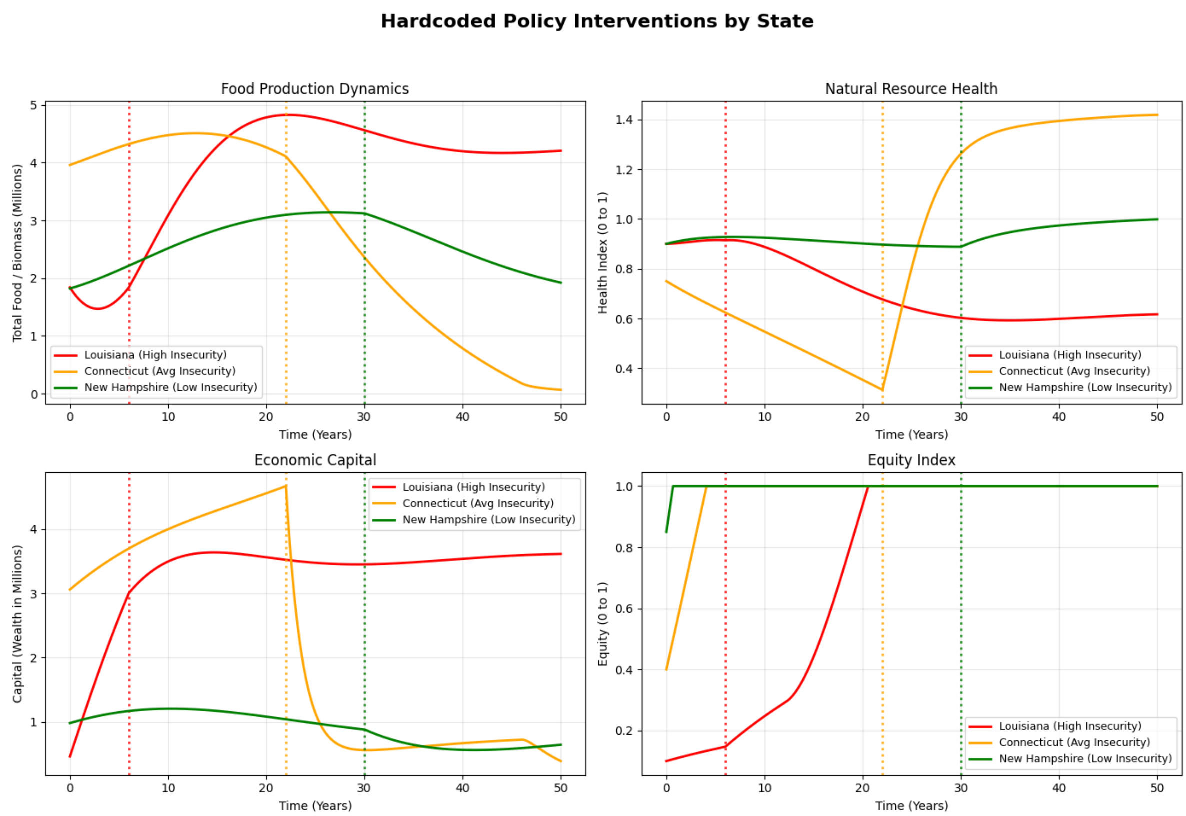

Figure 2.2: Simulation of the four state variables under Political Delay across Louisiana, Connecticut, and New Hampshire.

High: After policy implementation, food production immediately rises, reaching a peak a little after 20 years. Production then slowly declines to just above 4 million units. Environmental health shows a decline but remains at a sustainable level. After a steep increase without the policy, capital stays between 3 and 4 million as funds are diverted to promoting environmental health and equity. The slope of the equity curve increases immediately after the policy, then becomes steeper until reaching 1.0.

Average: Food production declines after policy introduction until eventually reaching just above 0. Economic capital suffers an immediate steep decline and remains low. Although equity is maintained, there is no food to share, so everyone has equal access to very little food. Environmental health flourishes beyond realistic expectations without immense food production draining natural resources.

Low: No policy previously triggered, so we observe a preventative policy that ensures long-term environmental preservation and equity. Food production declines, but it still produces enough for the population. The already high natural health index increases to 1.0, and equity remains at 100%. Capital declines, but does not significantly differ from the previous model with no intervention.

Chapter 3

Spatial Considerations and SPDE Modeling

While the foundational models in previous chapters have successfully captured the temporal dynamics, in this case, the real-world food systems are spatially heterogeneous. For instance, production, wealth, and ecological health do not distribute uniformly across a map. To account for the physical supply chains as well as the regional market shocks, we can transition the framework from Ordinary Differential Equations (ODEs) to Stochastic Partial Differential Equations (SPDEs). This transition will allow us to model the continuous flow of food and capital all across a geographic space while subjecting the system to environmental volatility.

3.1 The Spatial Logistics Framework

Starting off with a spatial dimension requires us to separate the variables that move from those that remain fixed to the ground. For instance, food (F) and economic capital (C) are highly mobile, with trucks transporting them all across highways as well as the fact that banks wire them across the borders. On the other hand, the Natural Resource Health (N) and the Equity Index (Eq) remain strictly localized. Soil degradation does not travel, and social inequality will remain tied to the specific population experiencing it.

To be able to model the physical distribution of goods from the highly efficient rural regions to high-demand urban centers, we introduce the Laplacian operator (∇^2) to the food and capital equations, which will simplify to the second spatial derivative ($\frac{\partial^2}{\partial x^2}$) throughout a one-dimensional supply chain.

$$dF = \left(\alpha C N u_p - \min(\beta P, F) - \gamma F + D_F \frac{\partial^2 F}{\partial x^2}\right) dt + \sigma_F F dW_F \tag{5}$$

$$dC = \left(p_{margin}(\min(\beta P, F)) - cF - (u_s + u_e)C + D_C \frac{\partial^2 C}{\partial x^2}\right) dt + \sigma_C C dW_C \tag{6}$$

The parameters D_F and D_C represent the diffusion coefficients of the supply chain logistics as well as the financial trade. A high D_F can supply a region with excellent infrastructure, allowing surplus food to flow smoothly into surrounding areas. On the other hand, having a low D_F will isolate a region, mathematically creating a physical food desert where the supply chain ends up breaking down. Furthermore, the capital generally diffuses in the opposite direction of food, where the urban centers import agricultural goods as their wealth flows back into the rural regions.

3.2 Stochastic Volatility and Market Shocks

Agriculture remains vulnerable to unpredictable environmental and economic shocks consisting of localized droughts, floods, or sudden trade embargoes. For us to model this volatility, we introduce a space-time Gaussian white noise term to the differential equations, represented by $\dot{W}(x,t)$. By applying a multiplicative noise term, $\sigma_F F \dot{W}(x,t)$, this model calculates the proportional risk. An agricultural hub producing massive amounts of biomass will suffer an even higher absolute loss during any event, such as extreme weather situations, compared to a low-producing region. This stochastic element proves that even a perfectly balanced deterministic system can, in fact, fall into some sort of crisis from a sudden, random spatial shock.

3.3 Boundary Conditions and Numerical Implementation

To computationally solve this SPDE system, we define the physical domain Ω and provide the Neumann (zero-flux) boundary conditions at the borders ($\partial\Omega$). This boundary assumes there to be a closed national economy where the food and capital can freely diffuse internally; however, they cannot leak out or enter from beyond the grid.

Since there are no analytical solutions to SPDEs in this format, we utilize the Method of Lines. We classify the continuous spatial landscape into a finite one-dimensional grid, as this represents a primary supply chain route. We also define the continuous 100-mile highway ($L = 100$) as a finite grid of $nx = 100$ nodes, as this is connected by a physical spatial step size dx:

$$dx = \frac{L}{nx - 1}\ . \tag{7}$$

Then to evaluate the flow of food across this matrix, we establish an initial condition that is precisely modeled by a Gaussian distribution. This makes sure that the food starts off highly concentrated at a specific geographic center point and remains sparse at the urban edges:

$$F(x,0) = 20 + 80 \exp\left(-0.05\left(x - \frac{L}{2}\right)^2\right)\ . \tag{8}$$

On the other hand, the economic capital is established with an inverted Gaussian distribution, representing high wealth concentration at the urban boundaries ($x = 0$ and $x = 100$) and a lower baseline in the agricultural center:

$$C(x,0) = 80 - 60\exp\left(-0.05\left(x - \frac{L}{2}\right)^2\right) . \tag{9}$$

To then advance the system through time, we apply the Euler-Maruyama method. In this case, this numerical update for any interior node j at time step i heavily relies on the total estimation of the continuous Laplacian operators for both state variables by using the finite central difference method:

$$\text{Laplacian}_{F,j} = \frac{F_{j+1}^{i-1} - 2F_j^{i-1} + F_{j-1}^{i-1}}{dx^2} \tag{10}$$

$$\text{Laplacian}_{C,j} = \frac{C_{j+1}^{i-1} - 2C_j^{i-1} + C_{j-1}^{i-1}}{dx^2} . \tag{11}$$

Finally, we calculate the deterministic drift as well as the stochastic diffusion from the previous time step. Here, the boundary conditions have strictly enforced a zero gradient at the edges to maintain the closed economy:

$$\text{Drift}_{F,j} = \alpha C N u_p - \min(\beta P, F_j^{i-1}) - \gamma F_j^{i-1} + D_F \text{Laplacian}_{F,j} \tag{12}$$

$$F_j^i = \max\left(0, F_j^{i-1} + \text{Drift}_{F,j} \cdot dt + \sigma_F F_j^{i-1} dW_F\right) . \tag{13}$$

Similarly, the discrete numerical update for Economic Capital relies on its own localized drift and stochastic market fluctuations (dW_C):

$$\text{Drift}_{C,j} = p_{margin} \min(\beta P, F_j^{i-1}) - cF_j^{i-1} - (u_s + u_e)C_j^{i-1} + D_C \text{Laplacian}_{C,j} \tag{14}$$

$$C_j^i = \max\left(0, C_j^{i-1} + \text{Drift}_{C,j} \cdot dt + \sigma_C C_j^{i-1} dW_C\right) . \tag{15}$$

Example: The 1D Highway Route

We can evaluate this spatial mathematics in practice by defining an agricultural ”breadbasket” exactly in the middle of this 100-mile highway ($x = 50$). In this case, at time zero, the Gaussian initial condition places a massive concentration of food at this center point. From there, we set the supply chain infrastructure coefficient to a highly efficient $D_F = 5.0$, as this means that the Laplacian operator will actively pull the food away from the center towards the starving urban boundaries. On the other hand, we introduce a severe weather volatility factor of $\sigma_F = 0.15$. The localized population demands food at a constant rate, and the system utilizes maximum production effort ($u_p = 1.0$) to continuously generate new biomass right at the center. By putting these parameters into the discrete numerical updates, the computer simulates the physical spread of agricultural wealth. Concurrently, the model establishes wealthy urban hubs at the boundaries ($x = 0$ and $x = 100$) with a capital diffusion coefficient of $D_C = 2.0$, allowing financial wealth to flow inward to pay for the agricultural imports, subjected to a market volatility of $\sigma_C = 0.05$.

3.3.1 Food System Logistics: Computational Results and Discussion

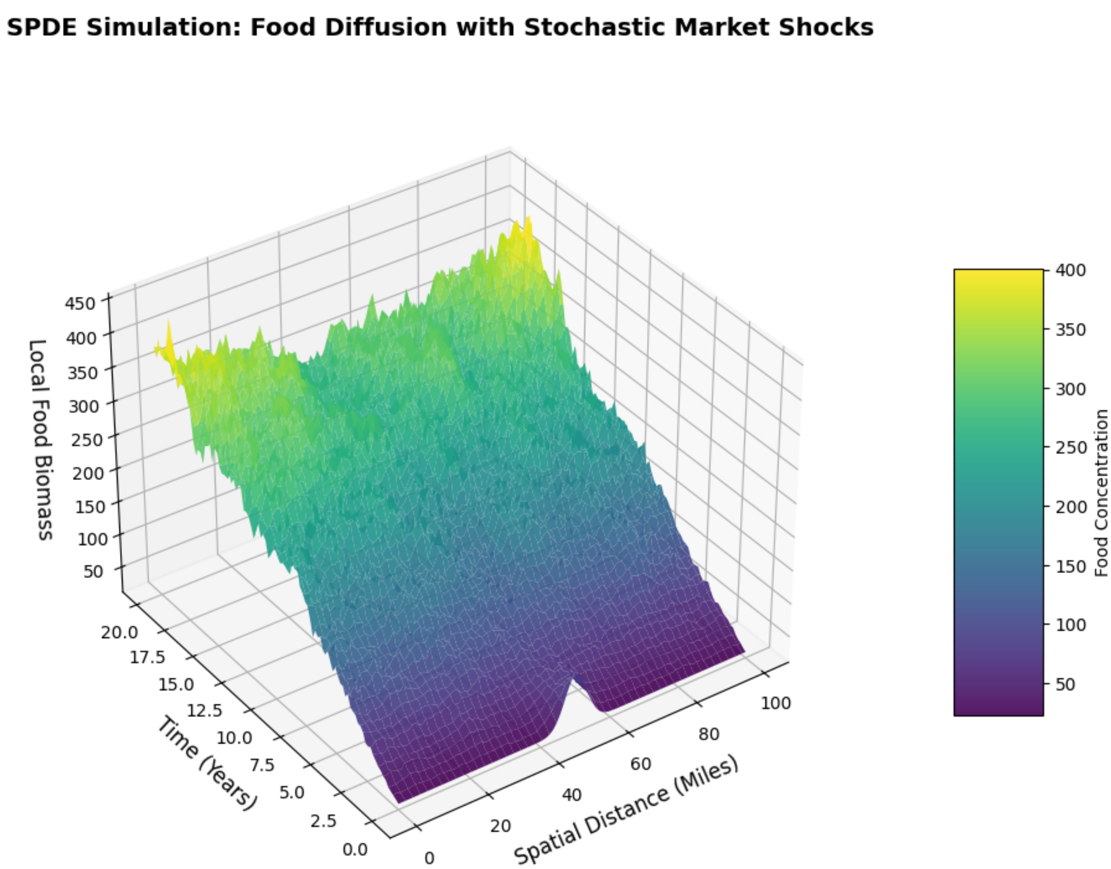

Figure 3.1: A 3D surface plot simulating Stochastic Partial Differential Equations over 20 years. The vertical axis represents the localized concentration of food, the depth axis tracks time, and the horizontal axis represents the 100-mile physical highway distance.

This three-dimensional surface plot shows us the key difference between the static ODEs and spatial SPDEs. At year zero, we can see that the food supply forms a sharp, distinct peak in the center of the domain, representing the isolated rural hub. As time progresses along the depth axis, the high diffusion coefficient (D_F) ends up acting upon the Laplacian operator, physically spreading the food outward toward the urban boundaries. On the other hand, the steep central mountain flattens into a broader structure, proving that the efficient supply chain infrastructure has naturally mitigated the localized food deserts.

However, we can also observe that the surface does not remain smooth. The multiplicative Gaussian white noise appears to introduce severe, localized ripples all across the food supply over the course of time. These ripples all represent the sudden agricultural shocks destroying crops before the trucks can even distribute them. Since the noise term scales proportionally with the available biomass ($\sigma_F F$), the volatility strikes the central breadbasket the hardest. Furthermore, this model proves that while spatial diffusion successfully distributes the resources, this entire interconnected network remains highly susceptible to regionalized, random weather events, requiring the governments to maintain emergency surplus reserves.

3.3.2 Economic Capital: Computational Results and Discussion

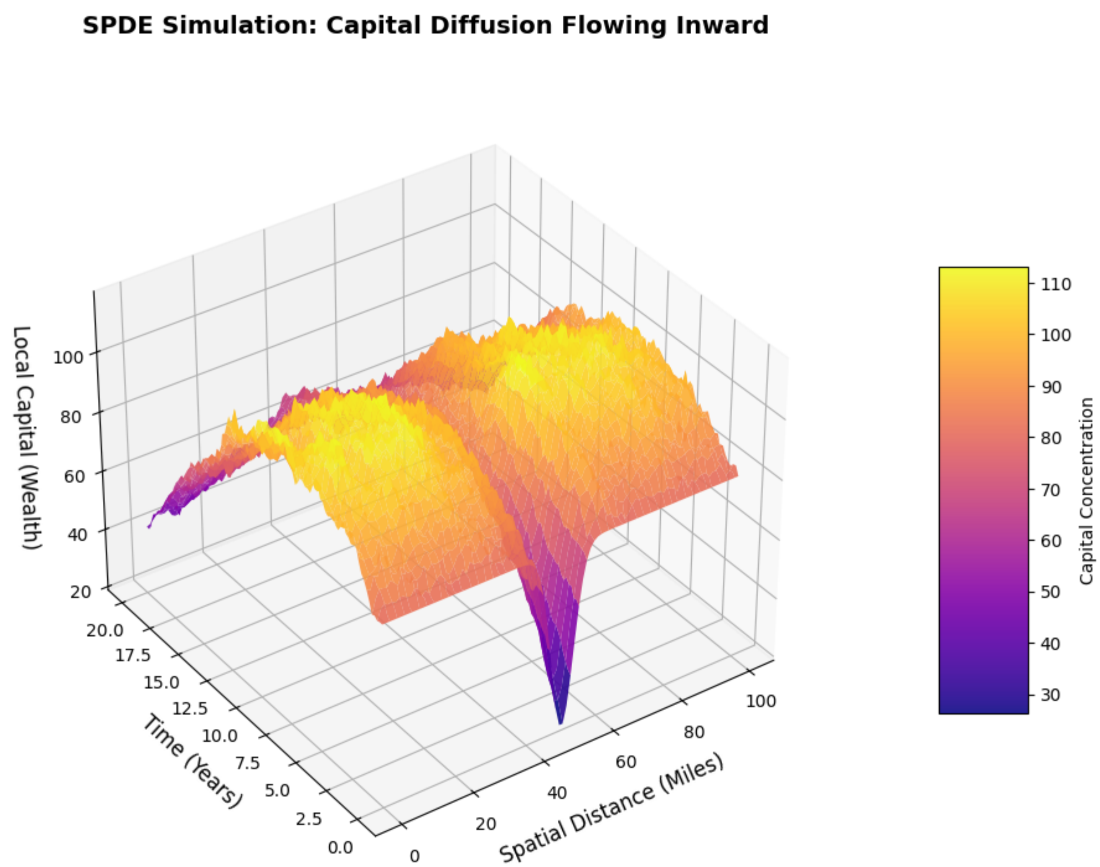

Figure 3.2: A 3D surface plot simulating Economic Capital diffusion. The surface demonstrates wealth flowing from the high-concentration urban boundaries inward toward the rural agricultural center over 20 years.

Similarly, the Capital SPDE visualizes the financial counterpart to the physical supply chain. At year zero, wealth is heavily concentrated at the urban boundaries, forming steep ridges on the edges of the domain. As food flows outward from the center, capital successfully diffuses inward to pay for the agricultural biomass. Over time, the capital distribution smooths out into a more uniform landscape, visually proving the economic interdependence between rural producers and urban consumers. However, much like the agricultural shocks, the capital surface exhibits its own high-frequency ripples ($\sigma_C C dW_C$), representing localized economic volatility and unpredictable market pricing.

Conclusion

Although the Python model gives significant insight into the dynamics of food production, natural resource health, economic capital, and equity, it cannot solve all the problems of our food system. With the immediate intervention models, we appear to reach a steady state of environmental health exactly at the set threshold, but the perceived equilibrium is a result of the rapid back-and-forth between the environmental policy and the balanced state. In the real world, governments could not constantly pass such legislations at every shift in the health index. In the delayed political intervention models, we use one policy for each state based on the first emergency threshold reached. For Louisiana's food crisis and New Hampshire's preventative measures, one policy can keep them from entering another state of emergency. Connecticut, however, quickly suffers a lack of food production after making efforts to preserve its natural health. It would require another policy, a lifetime clause, or some other intervention to correct the issue. Every food system on every level may be modeled and predicted, but real improvement requires government action and the participation of the people.

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Python Model Code

This section provides the core Python script that was used to model the general differential equations simulated in this report.

General ODE Models

```python

#General Models
import numpy as np
import matplotlib.pyplot as plt

# Define the system parameters
alpha = 0.35
beta = 0.05
gamma = 0.02
P = 100
r = 0.2
K = 1.0
delta = 0.001
sigma = 0.1
p_margin = 2.5
c = 0.05
k1 = 0.8
k2 = 0.005
D = 80
mu = 0.05

# Setup Custom Integration (Euler Method)
dt = 0.01  # DECREASED time step size for better mathematical stability
t = np.arange(0, 50, dt)  # Time array from 0 to 50 years

# Create empty arrays to store the results
F = np.zeros(len(t))
N = np.zeros(len(t))
C = np.zeros(len(t))
Eq = np.zeros(len(t))

```

```
    # Set initial conditions at index 0
    F[0] = 100.0
    N[0] = 0.9
    C[0] = 30.0
    Eq[0] = 0.3

    # Run the simulation loop step-by-step
    for i in range(1, len(t)):
        # Determine the policy levers for the current time
        current_time = t[i-1]

        up, us, ue = 1.0, 0.01, 0.01

        # Get the previous values
        prev_F = F[i-1]
        prev_N = N[i-1]
        prev_C = C[i-1]
        prev_Eq = Eq[i-1]

        # 1. Calculate how much food can actually be sold
        # It's either the full population demand, or whatever food is left, whichever is smaller.
        actual_sales = min(beta * P, prev_F)

        # 2. Update derivatives (using actual_sales instead of beta * P for revenue)
        dF = alpha * prev_C * prev_N * up - actual_sales - gamma * prev_F
        dN = r * prev_N * (1 - prev_N / K) - delta * prev_F * up + sigma * us
        dC = p_margin * actual_sales - c * prev_F - (us + ue) * prev_C
        dEq = k1 * ue + k2 * max(0, prev_F - D) - mu * prev_Eq

        # Update each state variable SEPARATELY
        # Added max(0, ...) to prevent unphysical negative values!
        F[i] = max(0, prev_F + (dF * dt))
        N[i] = max(0, prev_N + (dN * dt))
        C[i] = max(0, prev_C + (dC * dt))
        Eq[i] = min(1.0, max(0, prev_Eq + (dEq * dt)))

    # 5. Generate the 4 separate plots
    plt.figure(figsize=(8, 4))
    plt.plot(t, F, color='blue', linewidth=2)
    plt.title('Food Production Dynamics Over Time')
    plt.grid(True, alpha=0.3)
    plt.xlabel('Time (Years)')
    plt.ylabel('Total Food / Biomass')
    plt.show()
```

```python

    plt.figure(figsize=(8, 4))
    plt.plot(t, N, color='green', linewidth=2)
    plt.title('Natural Resource Health Over Time')
    plt.grid(True, alpha=0.3)
    plt.xlabel('Time (Years)')
    plt.ylabel('Health Index (0 to 1)')
    plt.show()

    plt.figure(figsize=(8, 4))
    plt.plot(t, C, color='gold', linewidth=2)
    plt.title('Economic Capital Over Time')
    plt.grid(True, alpha=0.3)
    plt.xlabel('Time (Years)')
    plt.ylabel('Capital (Wealth)')
    plt.show()

    plt.figure(figsize=(8, 4))
    plt.plot(t, Eq, color='purple', linewidth=2)
    plt.title('Equity Index Over Time')
    plt.grid(True, alpha=0.3)
    plt.xlabel('Time (Years)')
    plt.ylabel('Equity (0 to 1)')
    plt.show()
```

Listing 3.1: General ODE Models